1. (30 pts, 8, 11, 11) **Core Topic: Differentiation:** The following problems are not related.

   (a) Find the derivative of $f(x) = \dfrac{\sec(x^2)}{ax + 7}$ where a is a constant. Do **Not** simplify your final answer.

   (b) Find the equation of the tangent line of $x^2 + xy \quad y^3 = 7$ at the point $(\ \ 2; \ \ 1)$.

   (c) Determine the $(x; y)$-coordinates where $y = x^2 \ p\overline{2 \quad x}$ has horizontal tangent lines.

2. (35 pts, 13, 11, 11) The following problems are not related.

   (a) Consider

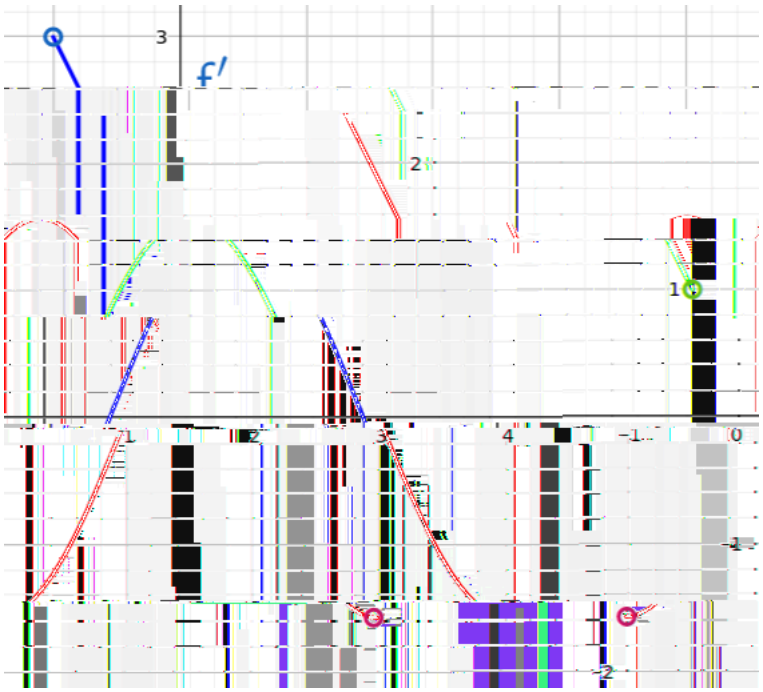
   $$g(x) = \begin{cases} p\overline{x} + 2 & 0 \quad x \quad 4 \\ x & x > 4 \end{cases}.$$

   Use the limit definition of the derivative to explain why the function g does not have a derivative when $x = 4$. (Explanations that do not use the limit definition of the derivative will earn no credit here.)

   (b) Consider a runner whose position function is $s(t) = t^2 + 5t + 50$ feet. Suppose a measurement of time $t = 10$ seconds is taken, with a possible error in measurement of up to 0:1 seconds. If the measured value is subsequently used to calculate the value of $y = s(10)$, use differentials to estimate the corresponding absolute error and relative error in the calculated y value.

   (c) Verify that the function $f(x) = x^3 \quad 6x$ satisfies the hypotheses of the Mean Value Theorem on the interval $[0; 3]$, and find all numbers c that satisfy the conclusion of that theorem.

3. (20 pts, 4 each) f is a continuous function on $[\ \ 1; 4]$ and the following graph corresponds to f'. Answer the following questions. No justification is required.



(a) On which intervals is f increasing?

(b) For what x-values in $(-1, 4)$ is f not differentiable?

(c) For what x-values in $(-1, 4)$ does f have a local maximum? For what x-values in $(-1, 4)$ does f have a local minimum?

(d) On what interval(s) is f concave down?

(e) Assume $f(0) = 0$. Sketch a graph of a function f that corresponds to this graph of $y = f'(x)$ and your answers from above. Clearly label the x-coordinates of all local maximums and minimums and inflection points. (There are many correct solutions.)

4. (15 pts, 7, 8)

Rosencrantz and Guildenstern were riding together on their way visit an old friend when the road diverged into two paths. Rosencrantz had a bike and took a road going North at 8 mph. Guildenstern, on the other hand, had a motorcycle and took a road going East at 15 mph.

(a) Draw a picture illustrating the situation taking care to label all variables. Write down an equation(s) that relates all the variables.

(b) How fast is the distance between them changing after an hour?

END OF TEST