

Exam 3**Instructions:**

- This exam has four problems on pages numbered 1 through 9. Make sure you have all pages.
- Write your name and section number at the top of each page.
- Show all work and simplify your answers, except where the instructions tell you to leave your answer unsimplified.
- Name any theorem that you use and explain how it is used.
- Answers with no justification will receive no points unless the problem explicitly states otherwise.
- Notes, your text and other books, calculators, cell phones, and other electronic devices are not permitted, except as needed to upload your work.
-
-

1. (40 pts)

(a) Evaluate the integral if it exists.

i. $\int \frac{3x + 9}{x^2 + 6x} dx$

ii. $\int_{\frac{1}{2}}^1 2 \sin(v) \cos(v) dv$

iii. $\int_3^3 (2x^4 + 3x) dx$

(b)

$$\begin{aligned}
 \int_{\frac{3}{4}}^{\frac{3}{4}} \cos(2x) \, dx &= \left[\frac{1}{2} \sin(2x) \right]_{\frac{3}{4}}^{\frac{3}{4}} \\
 &= \frac{1}{2} [\sin(\frac{3}{2}) - \sin(\frac{3}{2})] \\
 &= 0
 \end{aligned}$$

(c) Since g is odd $\int_{-5}^5 g(x) \, dx = 0$.

$$\begin{aligned}
 \int_{-5}^{-2} g(x) \, dx + \int_{-2}^0 g(x) \, dx + \int_0^5 g(x) \, dx &= 0 \\
 \int_{-5}^{-2} g(x) \, dx + 8 - 6 &= 0 \\
 \int_{-5}^{-2} g(x) \, dx &= -2
 \end{aligned}$$

Name _____

Section _____

2. (12 pts)

A fence is to be built to enclose a rectangular area of 250 square feet. The fence along three sides is to be made of material that costs 6 dollars per foot. The material for the fourth side will cost 10 dollars per foot. Find the dimensions of the enclosure that minimize the cost of fencing material.

Solution:

Let x be the width and y be the length of the enclosed area, and suppose that one of the sides of length y costs \$10. The two equations we have are:

$$A = xy = 250 \tag{1}$$

$$C = 6(2x)$$

3. (24 pts)

(a) Suppose an object moves with velocity $v(t) = 2t^2 - 12t + 16$ km/hr along a straight road.

i. Determine the displacement of the object on the time interval $[1,3]$.

ii. Determine the distance traveled on the time interval $[1,3]$.

(b) Apply Newton's method to the equation $x^3 + x - 5 = 0$: Use an initial guess of $x_0 = 1$ and find x_1 : (Find only x_1 .)

Solution: $x +$

(b) $f(x) = x^3 + x - 5$ and $f'(x) = 3x^2 + 1$.

$$\begin{aligned}x_1 &= x_0 - \frac{f(x_0)}{f'(x_0)} \\&= 1 - \frac{1 + 1 - 5}{3 + 1} \\&= 1 + \frac{3}{4} \\&= \frac{7}{4}\end{aligned}$$

Name _____

Section _____

4. (24 pts)

- (a) Evaluate the Riemann sum for $f(x) = x^2 - 3$ taking the sample points to be right endpoints, $a = -4$, $b = 2$ and $n = 6$.
- (b) Express the integral $\int_{-4}^2 (x^2 - 3) dx$ as a limit of Riemann sums. You are not required to fully simplify this expression.
- (c) Evaluate the expression that you gave in (b). Show all steps to find the limit of the Riemann sums.

Solution:

(a) With $a = -4$; $b = 2$; $n = 6$, $\Delta x = \frac{2 - (-4)}{6} = 1$. We make a table:

Name _____

Section _____

Name