Transport through chaos

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[illegible]

determine how ensembles of points are transported. The action principle of MacKay, Meiss and Percival [4] can be used to compute areas of pieces of the grid. Thus knowledge of trellis geometry together with area computations will form the

Lemma 2.1. Suppose that the curve is a piece of the stable manifold of some

Rearranging the sum gives

$$\int_D \mathrm{d}p \wedge \mathrm{d}q = \sum_{j=-\infty}^{\infty} \alpha^j [F(b_j) - F(a_j)].$$

In general suppose that D is a disc bounded by alternating segments of stable and unstable manifold. Suppose that the endpoints of these segments are indexed $a^0, a^1, \ldots, a^{2m}$ (with $a^{2m} = a_0$) in a counterclockwise order around the boundary of D. Suppose that the segment joining a^0 and a^1 is contained in a stable manifold. Then by the preceding argument

$$\int_D \mathrm{d}p \wedge \mathrm{d}q = \sum_{j=-\infty}^{\infty} \alpha^j \sum_{k=0}^{m-1} [F(a_j^{2k+1}) - F(a_j^{2k})]. \tag{2.4}$$

This formula expresses the MacKay–Meiss–Percival action principle.

The action function F can be easily calculated for a specific map f. For example suppose that f is the standard map given by $f(q, p) = (q + p + a \sin(2\pi q),\ p + a \sin(2\pi q))$. Then $\mathrm{d}F = p\, \mathrm{d}q - f * p\, \mathrm{d}q$ can be calculated:

$$\mathrm{d}F = [2\pi ap \cos(2\pi q) + a \sin(2\pi q) + \pi a^2 \sin(2\pi q) \cos(2\pi q)]\, \mathrm{d}q + [p + a \sin(2\pi q)]\, \mathrm{d}p.$$

Furthermore $\mathrm{d}F = F_q\, \mathrm{d}q + F_p\, \mathrm{d}p$ where F_q denotes the partial derivative $\partial/\partial qF$. Since formulae for the partial derivatives of F are known, F is obtained by integration. The end result is that

$$F(q, p) = \tfrac{1}{2}p^2 + ap \sin(2\pi q) - \cos(2\pi q)/2\pi a - \tfrac{1}{2}a^2 \cos(4\pi q).$$

[redacted]

Proposition. Discontinuity points of t^+ occur on R-stable manifolds. Similarly, discontinuity points of t^- occur on R-unstable manifolds. Hence the internal trellis of the resonance zone partitions the zone into its exit time decomposition.

[redacted]

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