

## Transport through chaos

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1991 Nonlinearity 4 583

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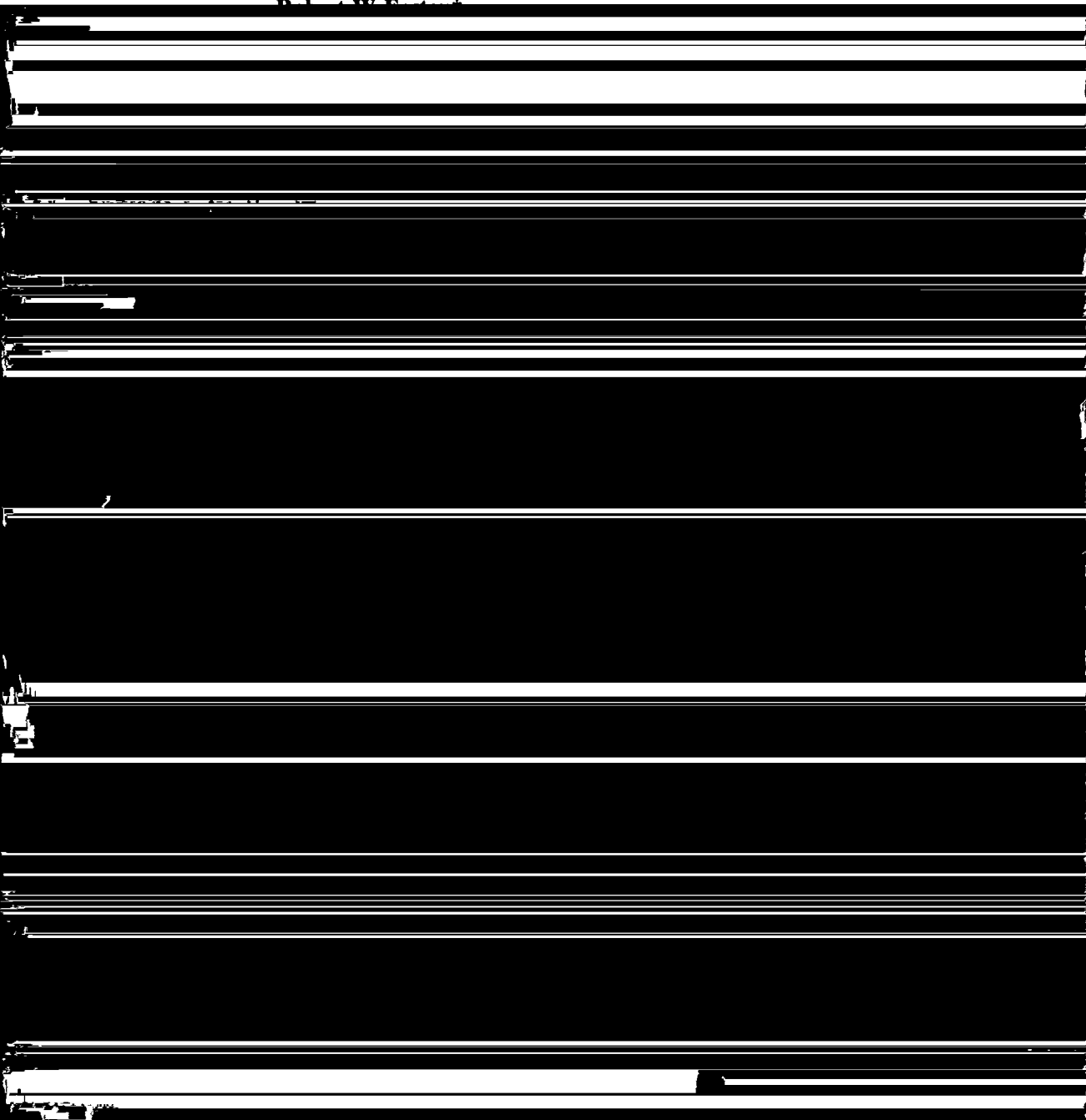
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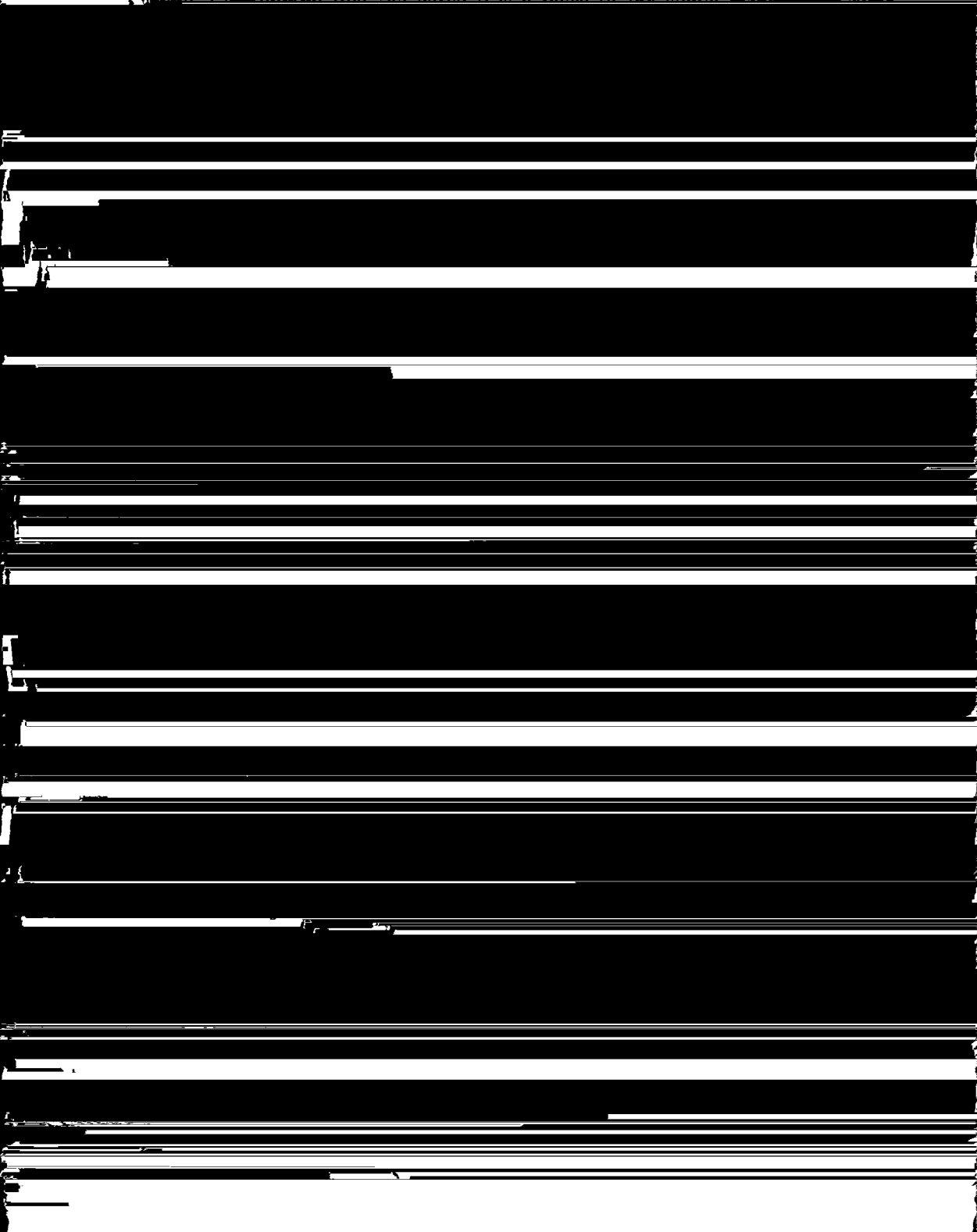
## Transport through chaos

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determine how ensembles of points are transported. The action principle of MacKay, Meiss and Percival [4] can be used to compute areas of pieces of the grid. Thus knowledge of trellis geometry together with area computations will form the

Figure 2.1. Suppose that the curve  $\mu$  is a piece of the stable manifold of some



Rearranging the sum gives

$$\int_D dp \wedge dq = \sum_{j=-\infty}^{\infty} \alpha^j [F(b_j) - F(a_j)].$$

In general suppose that  $D$  is a disc bounded by alternating segments of stable and unstable manifold. Suppose that the endpoints of these segments are indexed  $a^0, a^1, \dots, a^{2m}$  (with  $a^{2m} = a_0$ ) in a counterclockwise order around the boundary of  $D$ . Suppose that the segment joining  $a^0$  and  $a^1$  is contained in a stable manifold. Then by the preceding argument

$$\int_D dp \wedge dq = \sum_{j=-\infty}^{\infty} \alpha^j \sum_{k=0}^{m-1} [F(a_j^{2k+1}) - F(a_j^{2k})]. \quad (2.4)$$

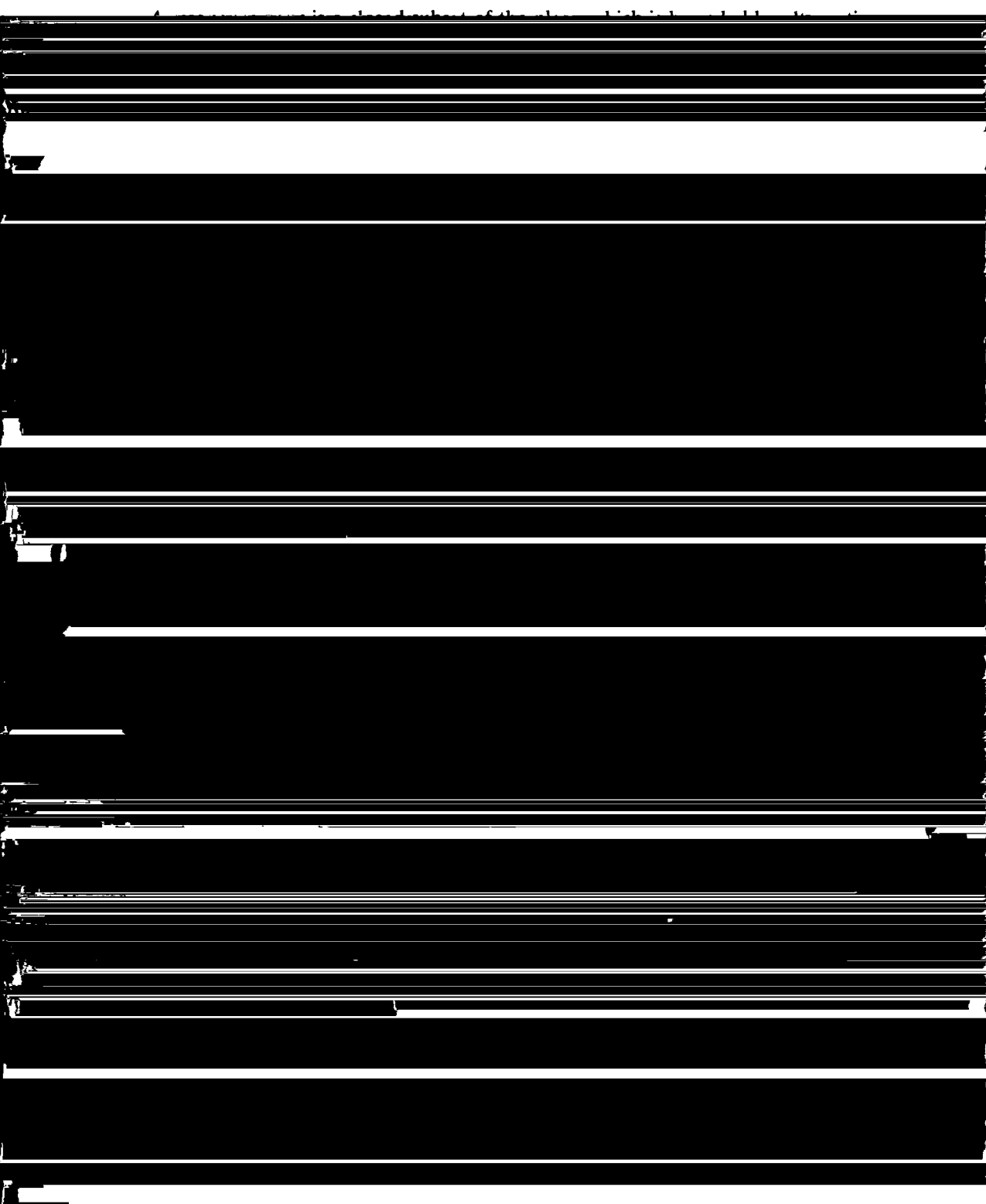
This formula expresses the MacKay–Meiss–Percival action principle.

The action function  $F$  can be easily calculated for a specific map  $f$ . For example suppose that  $f$  is the standard map given by  $f(q, p) = (q + p + a \sin(2\pi q), p + a \sin(2\pi q))$ . Then  $dF = p dq - f * p dq$  can be calculated:

$$dF = [2\pi a p \cos(2\pi q) + a \sin(2\pi q) + \pi a^2 \sin(2\pi q) \cos(2\pi q)] dq + [p + a \sin(2\pi q)] dp.$$

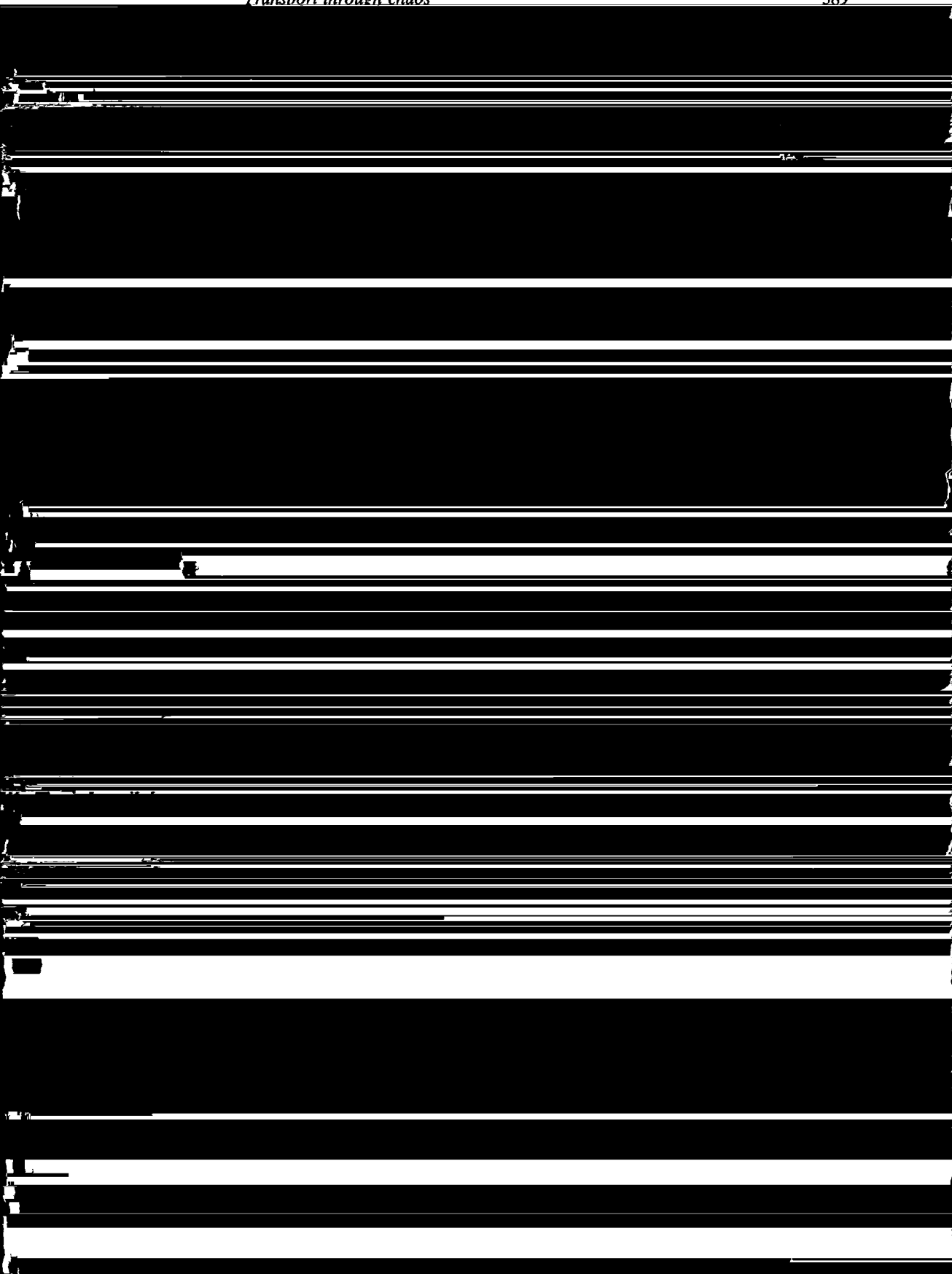
Furthermore  $dF = F_q dq + F_p dp$  where  $F_q$  denotes the partial derivative  $\partial/\partial q F$ . Since formulae for the partial derivatives of  $F$  are known,  $F$  is obtained by integration. The end result is that

$$F(q, p) = \frac{1}{2}p^2 + ap \sin(2\pi q) - \cos(2\pi q)/2\pi a - \frac{1}{2}a^2 \cos(4\pi q).$$



*Proposition.* Discontinuity points of  $t^+$  occur on  $R$ -stable manifolds. Similarly, discontinuity points of  $t^-$  occur on  $R$ -unstable manifolds. Hence the internal trellis of the resonance zone partitions the zone into its exit time decomposition.

*Proof.* For simplicity the proof will be given for the resonance zone pictured in





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